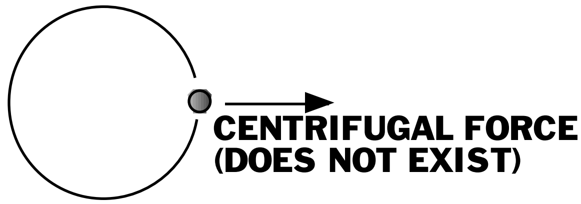
Loops

(Circular Motion, Potential Energy and Kinetic Energy)

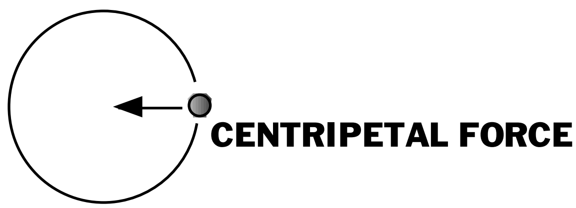
LOOPS

CENTRIPETAL FORCE

The average person on the street has heard of centrifugal force. When asked, they would describe this force as the one pushing an object to the outside of a circle. There is only one problem with this description. There is NO FORCE pushing an object to the outside.

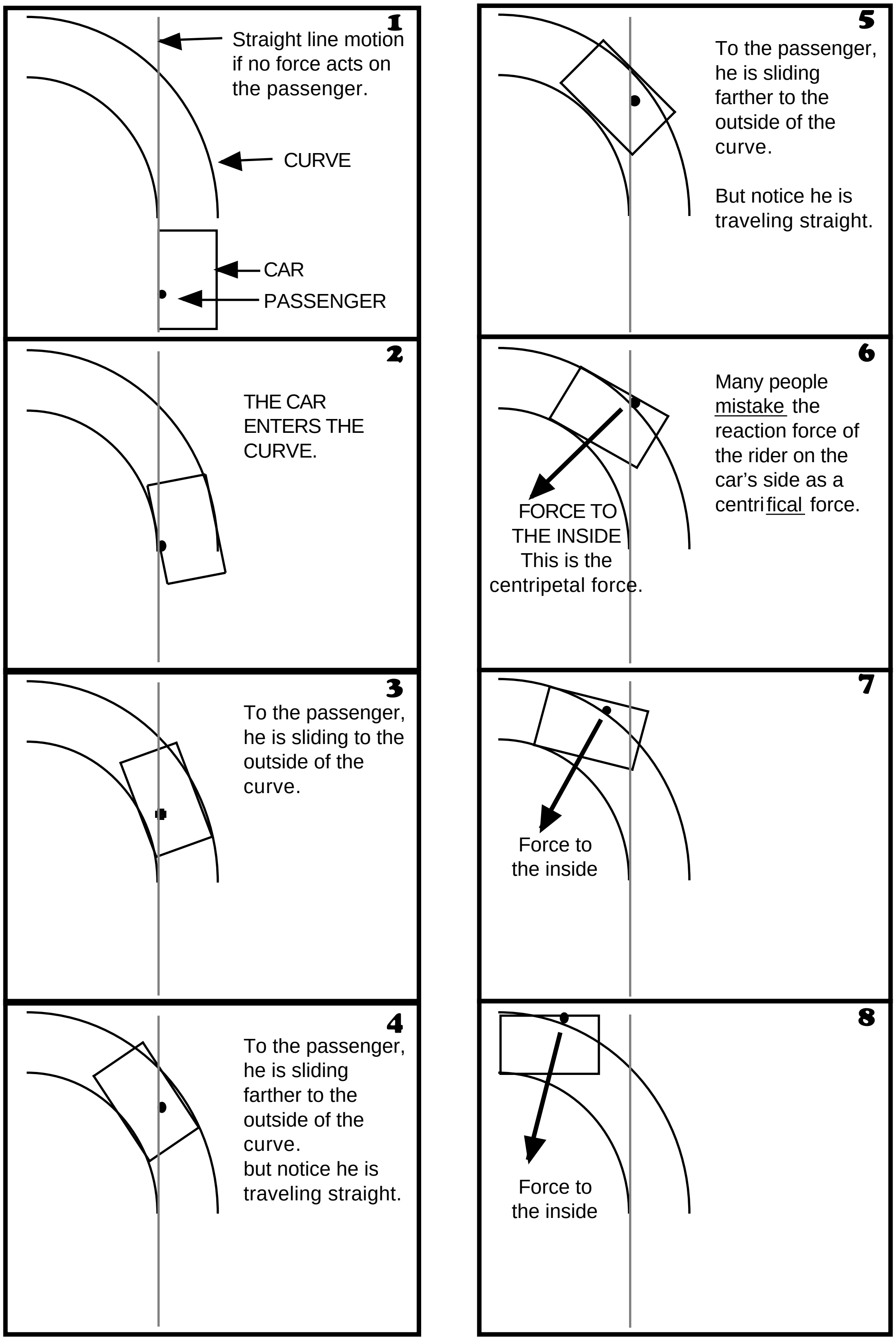


For a person riding in a car while traveling in a circle, he perceives a force pushing him to the outside of the circle. But what force is physically pushing him? It can not be friction. Frictional forces oppose the direction of motion. It can't be a "normal force[1] ." There is not a surface pushing the rider to the outside. To travel in a circle, a force pointing to the inside of the circle, or curve, is needed. The force pointing to the inside is called the *centripetal force*.



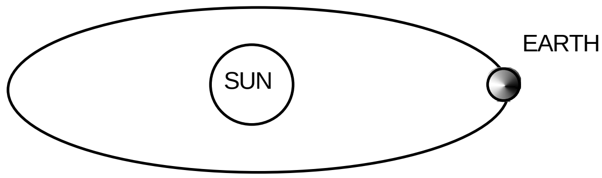
To understand a source for the misconception of the direction of this force, consider what it feels like when traveling around a corner in the back seat of a car. Everyone who has been in this situation knows that the passenger will slide to the outside of the curve. To understand that there is no force pushing the passenger to the outside, a change of reference frame is needed. Move the point of view from inside the car to a location outside, above, the car.

[1] The normal force is the force perpendicular to a surface. The floor is exerting a normal force straight up equal to your weight right now.


1
Straight line motion if no force acts on the passenger.
CURVE
CAR
PASSENGER
2
THE CAR ENTERS THE CURVE.
3
To the passenger, he is sliding to the outside of the curve.
4
To the passenger, he is sliding farther to the outside of the curve.
but notice he is traveling straight.
5
To the passenger, he is sliding farther to the outside of the curve.
But notice he is traveling straight.
6
Many people mistake the reaction force of the rider on the car's side as a centrifical force.
FORCE TO THE INSIDE This is the centripetal force.
7
Force to the inside
8
Force to the inside

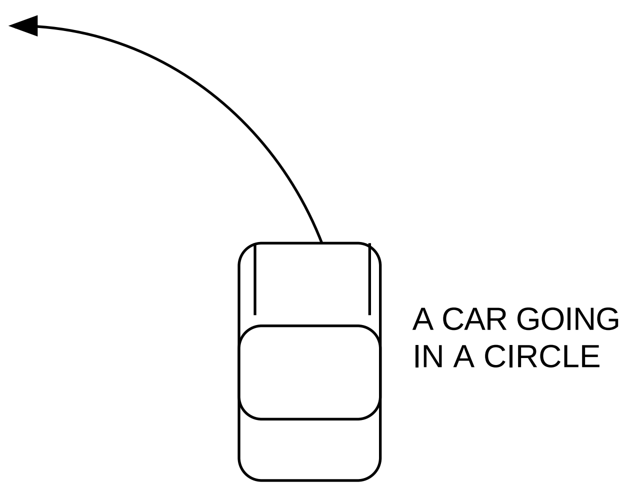
OVERHEAD

CENTRIPETAL FORCE AND WHAT SUPPLIES IT



Gravity supplies the centripetal force that keeps the Earth orbiting the sun.

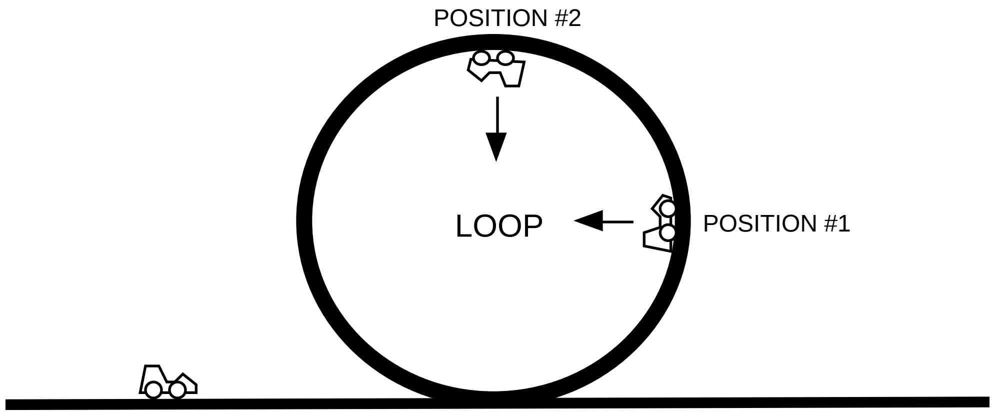
The pull of the person's arm and gravity -when the bucket is upside down- supplies the centripetal force.



Friction between the tires and the road supplies the centripetal force. If the road is banked, then gravity will contribute to the centripetal force.

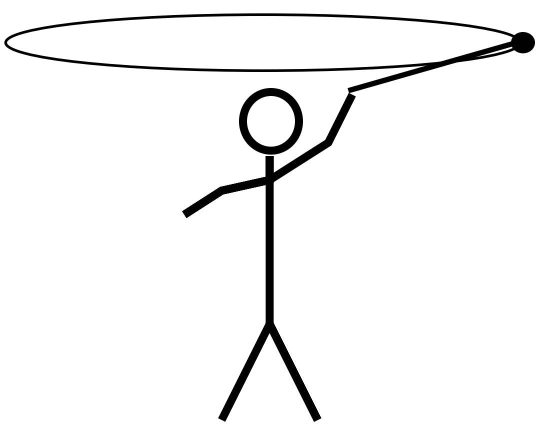
OVERHEAD

CENTRIPETAL FORCE AND WHAT SUPPLIES IT



The normal force the track exerts supplies the centripetal force at position #1.

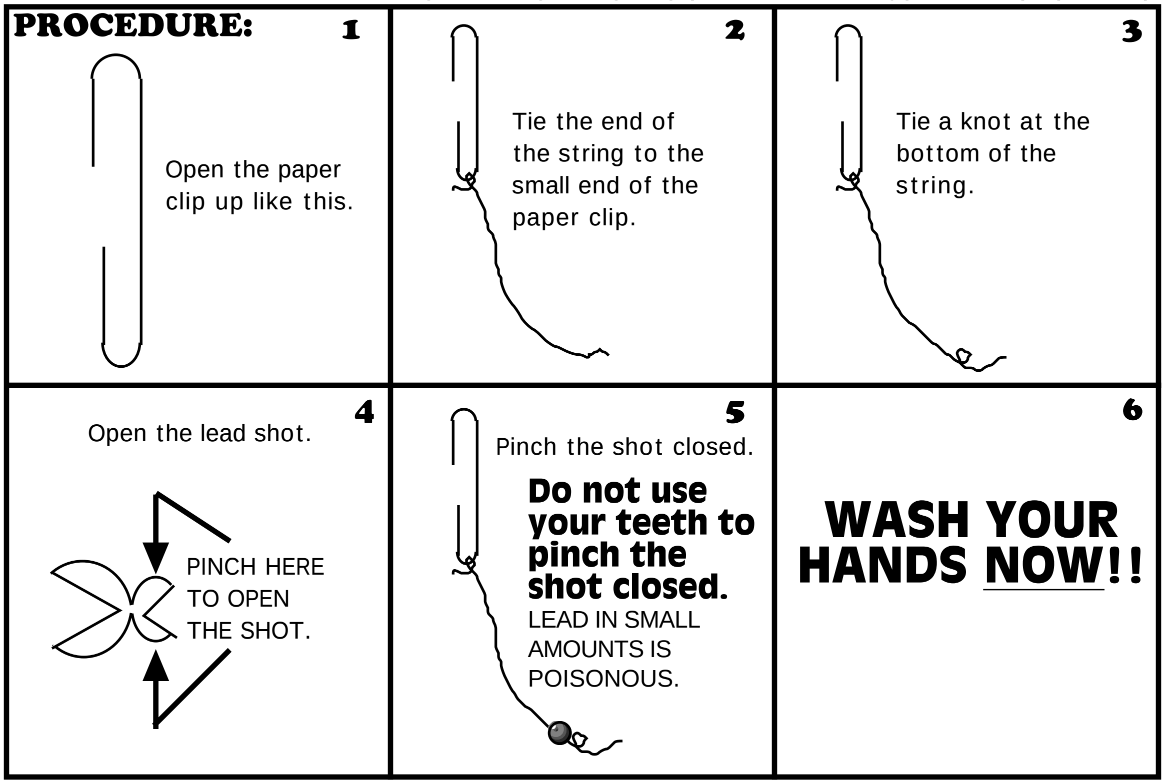
The normal force the track exerts plus the pull of gravity exerts the centripetal force at position #2



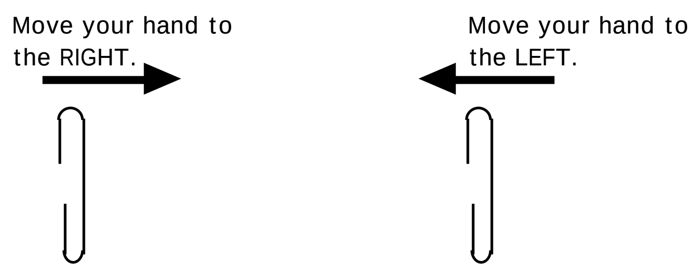
The string exerts the centripetal force on the rock.

ACTIVITY -ACCELEROMETER

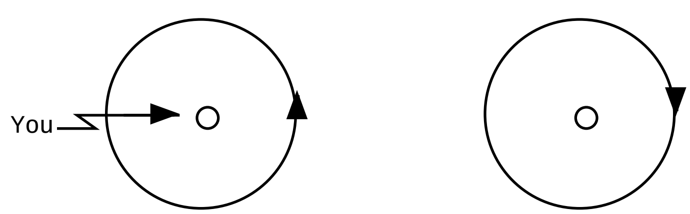
MATERIALS: 6 inches of string, fishing weight (split shot variety), small paper clip



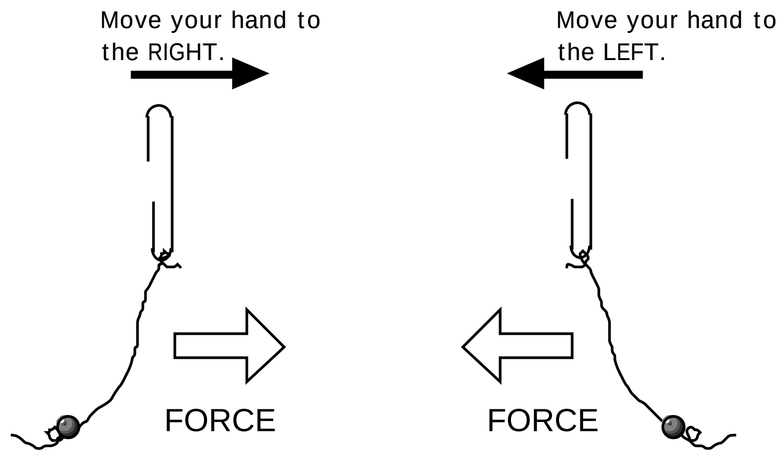
Hold the accelerometer by the paper clip in your hand. Move your hand to the right then to the left. Draw the <u>direction the accelerometer swings</u>. Also indicate the <u>direction of the force</u> as you move the accelerometer.

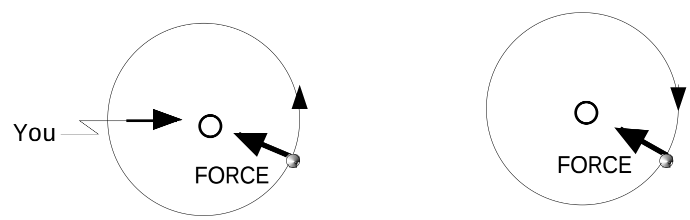


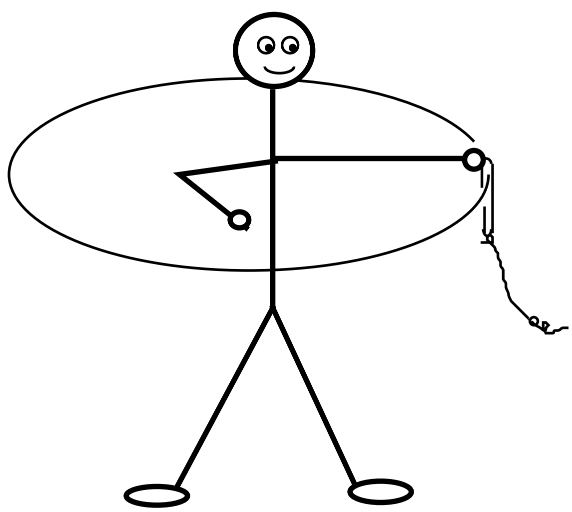
Hold the accelerometer in one hand. Hold your hand outstretched and twirl around while watching the accelerometer. Below is the circle your hand makes as viewed from overhead. Draw which way the <u>accelerometer swings</u>. Also label the <u>force's direction</u>.



ACTIVITY -ACCELEROMETER ANSWERS







STAYING IN YOUR SEAT WITH CENTRIPETAL FORCE

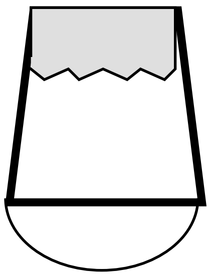
DEMO

MATERIALS:

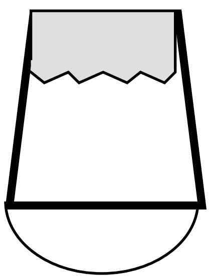
Bucket of water

PROCEDURE:

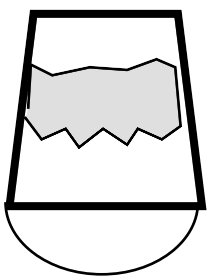
Fill the bucket up $^{1}/4$ full with water. Swing the bucket in a vertical circle. Swing it fast enough so the water does not come out. Now slow down the swing until the water almost drops out of the bucket.



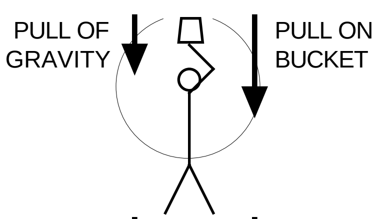
In order to keep water in the bucket when it is upside down, the water needs to be accelerated,(pulled downwards), faster than what gravity would move the water. The pulling force downward varies with the velocity the bucket moves. By spinning the bucket faster, the pull downwards is greater.



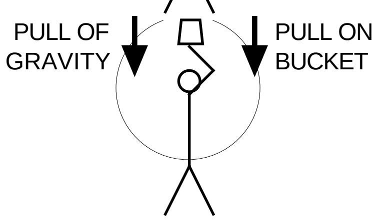
In order to just barely keep water in the bucket when it is upside down, the water needs to be accelerated, (pulled downwards), as fast as what gravity would move it. The pulling force downward varies with the velocity the bucket moves. The bucket needs to be twirled at a speed so that the pull equals the pull of gravity.



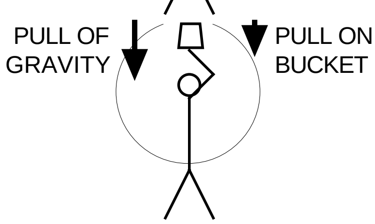
When the water falls out of the bucket, it is because the pull on the water is greater than the pull on the bucket. Gravity is accelerating the water. If swung slow enough the person's arm is resisting the fall of the bucket. Gravity moves the water out of the bucket. By pulling harder on the bucket, the bucket will be pulled down at an acceleration equal to or greater than what gravity produces. If the bucket is pulled down faster than what gravity pulls, the water stays inside.



When the pull on the bucket is greater than the pull of gravity, the water stays in the bucket.



When the pull on the bucket is equal to the pull of gravity, the water just barely stays in the bucket.



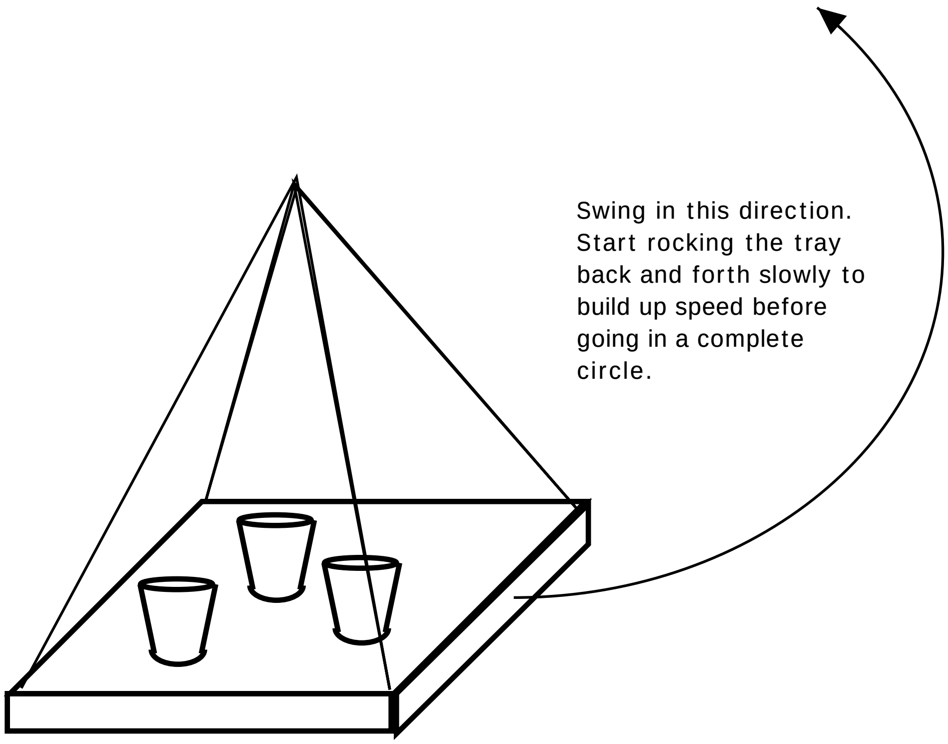
When the pull on the bucket is less than the pull of gravity, the water falls out of the bucket.

This same demo can be duplicated a couple different ways.

Materials:

Plastic cups filled 1/4 full with water, strong string, wood or cafeteria service tray

Procedure:

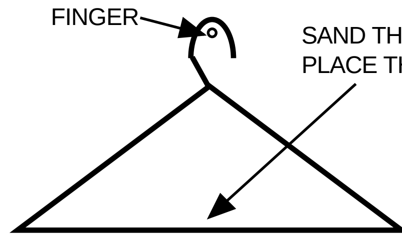


MATERIALS:

penny, metal coat hanger

PROCEDURE:

With a file, rub the inside bottom of the coat hanger flat. Balance a penny on the inside of the coat hanger while hanging the hanger on your index finger. Gently rock the hanger back and forth before swinging it all the way around.



The coat hanger supplies the centripetal force to the inside of the circle to keep it going in a circle.

The Most Common Questions and Answers About Centripetal Force

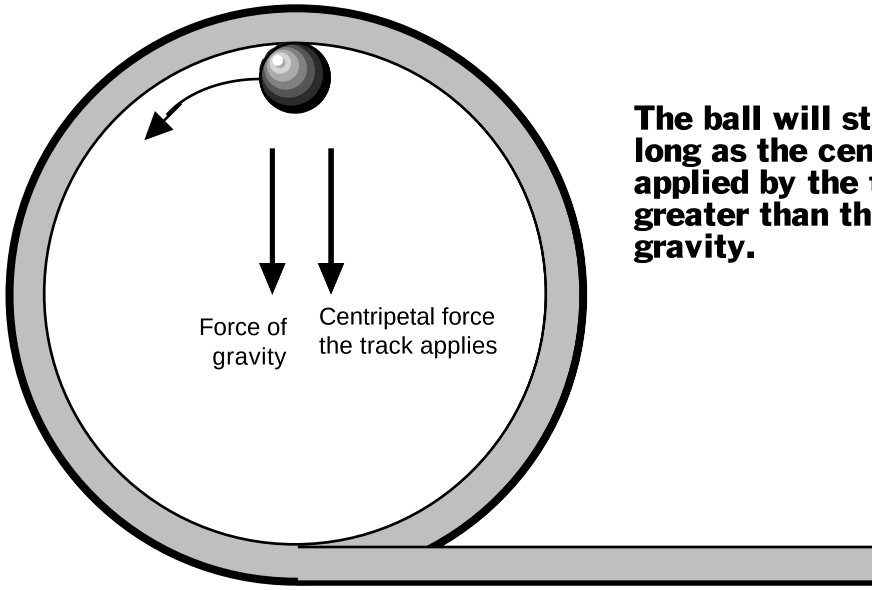
Q. "If I tie a rock onto the end of a string and swing it around my head in a horizontal circle, I feel the rock pulling my arm to the outside. This certainly feels like a centrifical force and not a centripetal force. What is going on here?"

A. The pull the person swinging feels is the pull to the inside against the inertia of the object spinning. Remember, in the absence of a force, a body will travel in a straight line. In order to turn the rock, a force to the inside must be applied. According to Newton's 3rd law of motion; for every action, there is an equal and opposite reaction.

THE PASSENGER GOES TO THE OUTSIDE DUE TO THE LACK OF ENOUGH CENTRIPETAL FORCE -NOT THE APPLICATION OF A CENTRIFUGAL FORCE. This is true for anything whose motion changes from a circle to a straight line.

A COASTER'S LOOP

On a well designed roller coaster loop, the riders will not be able to sense when they are traveling upside down. This is done by making sure the force that is exerted on the rider is at least equal to the weight of the rider.



The ball will stay on the track as long as the centripetal acceleration applied by the track is equal to or greater than the acceleration of gravity.

Centripetal force applied to the track depends on the velocity of the car and inversely to the radius. The formula is:

$$F = m\frac{V^2}{R}$$

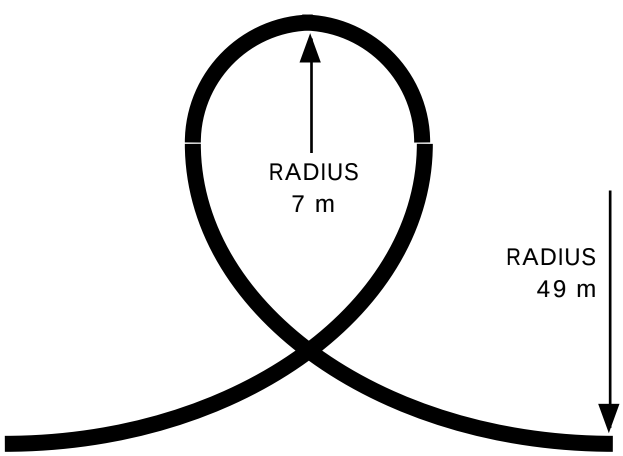
$$a_c = \frac{V^2}{R}$$

F = Centripetal force
m = mass of the object going in a circle
v = Object's velocity
R = Radius of circle of curve
a_c = centripetal acceleration

In order to apply enough centripetal acceleration the roller coaster car has to either be traveling very fast or the radius of the loop has to be made small. Most rides have a tall loop. A tall loop means a big radius. The problem is, as a car goes up, it slows down. The higher it goes, the slower it will be traveling over the top. In order to apply a centripetal force equal to gravity, 1 g, at the top, the car must be traveling extremely fast as the rider enters the loop. On some of the early round loops, the riders actually had their necks broken as a combination of the sudden rise in the loop as they entered at an extremely high rate of speed. As a compromise, the loops today are designed around an irregular shape called a klothoid or spiral of Archimedes. These irregular loops allow a circular figure whose radius changes.

"Klothoid" shaped loop from the Shock Wave at Paramount's Kings Dominion in Doswell, Virginia.

This is the Loch Ness Monster at Busch Gardens in Williamsburg, Virginia. It has two loops that are designed from the spiral of Archimedes. One loop is easy to identify in the picture. Can you spot the second loop?



Klothoid or Spiral of Archimedes Loop

The loop is tall and the radius is small. The smaller radius means the coaster car can travel slower and still apply enough force so the riders do not fall out.

For the advanced reader, the formula for the klothoid shape is:

$$x = \pm A \int_0^t \frac{\sin(t)\,dt}{\sqrt{t}} \qquad y = \pm A \int_0^t \frac{\cos(t)\,dt}{\sqrt{t}}$$

Asymptotic points: $(\pm A/2, \pm A/2)$

The formula for the "Spiral of Archimedes" in polar form is

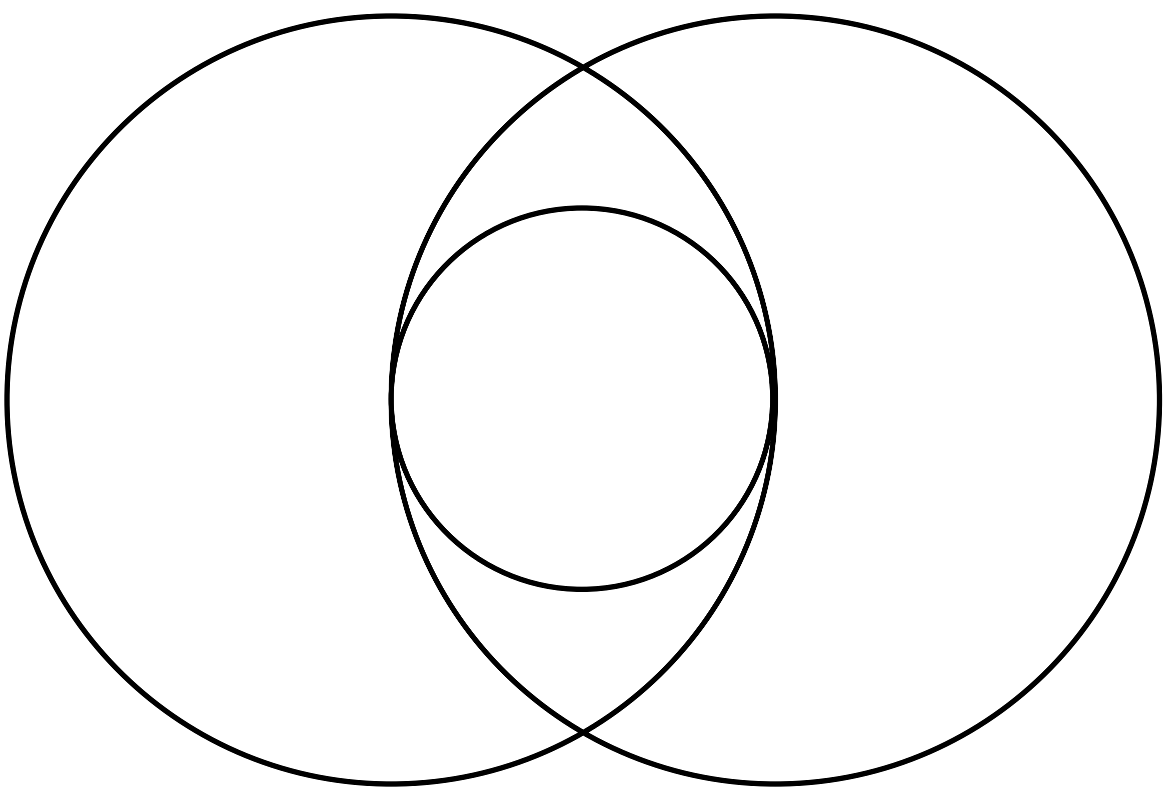
$$\mathbf{r = a}\theta$$

where "a" describes the magnitude of the spiral and "θ" is the angle through which the spiral is formed. To make a loop, the spiral will have to be mirrored horizontally.

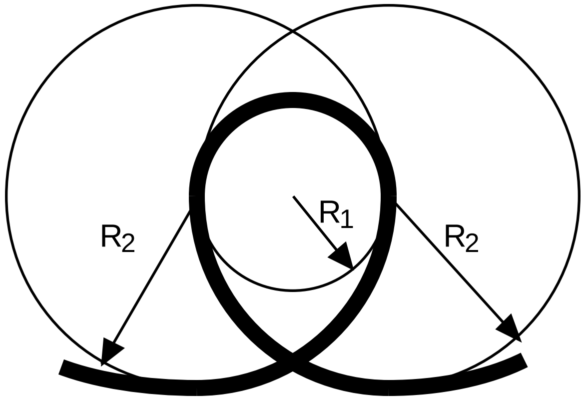
Nothing is perfect in engineering. These designs operate under ideal circumstances. In real life, the curves need to be tweaked into the right shape.

The Simple Irregular Loop

Sometimes it is not necessary to go into all the math to have a little fun with the irregular loop. These loops can be simulated using the combination of semi-circles of different radii.



Can you see the irregular loop in these regular circles?



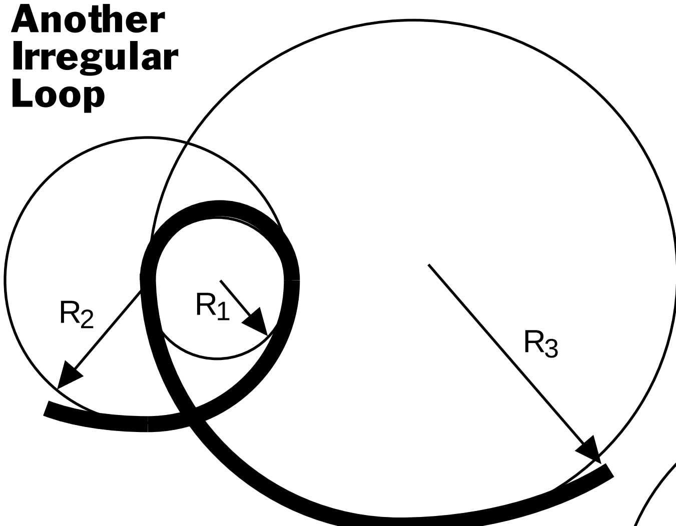
The radii can be anything as long as the car will make it around. In this particular drawing the height at the top of the loop from the very bottom is $(1/2)R_2 + R_1$.

If the engineer so chose, she could make the radius at the bottom on the way in one value and the bottom radius on the way out a different value.

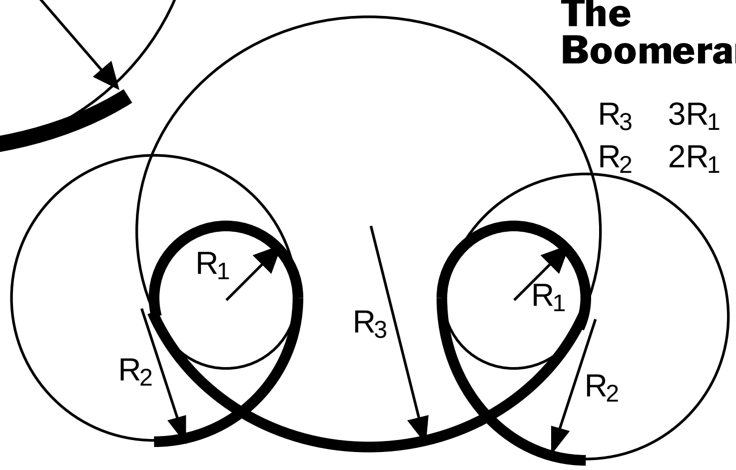
Do not design a real roller coaster with this method. The transition from different radii would be uncomfortable for the rider and not possible for the roller coaster train.

Other loop possibilities

Another Irregular Loop



The Boomerang

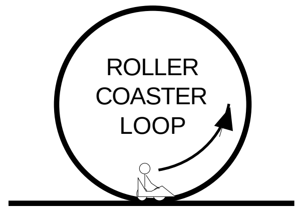


Physiological Effects of Acceleration

(Acceleration, Circular Motion, Kinetic Energy, and Potential Energy)

BACKGROUND

Imagine a passenger riding through a loop on a roller coaster. The passenger's head is towards the inside of the circle.



Her feet are to the outside of the circle. In order to keep blood in the passenger's head, a centripetal force needs to be applied to the blood to push it upwards toward the head and the center of the circle. The heart applies the centripetal force on the blood. A passenger can experience many g's in a loop. Recall that a g is the number of times heavier an object becomes. A 7 g experience means that the passenger feels 7 times heavier. Everything about the passenger becomes 7 times heavier. Her 3 pound brain now weighs 21 pounds. Every ounce of blood becomes 7 times heavier. If the blood feels too heavy the heart cannot apply enough force to push it towards the head. If the brain does not get any blood it will not get the oxygen the blood carries. The passenger will pass out within a second.

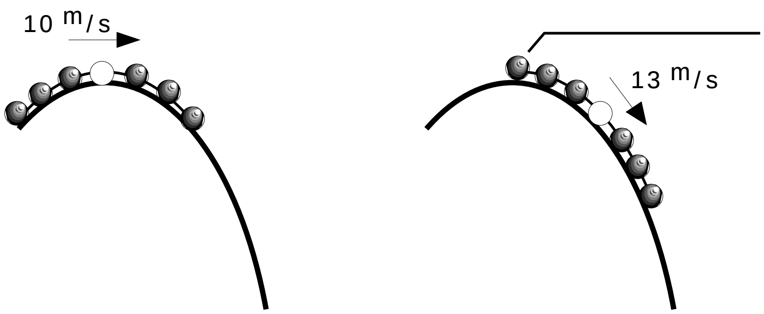
THE EXTREME EXPERIENCE

You are riding a new untested roller coaster when something goes wrong. As you enter the first big loop, a great pressure pushes you down. You slouch down in the seat from the extra weight. Over the top of the loop the roller coaster car slows down. The extra weight on your legs, lap, and shoulder make it impossible to sense that you are upside down. Out of the loop, over a hill and into another loop. This loop has a smaller radius. The car is traveling much faster now. As the g forces climb up toward 7 g's, you sink further still in the seat. You can no longer see color. Everything appears in black and white. An instant later, the passenger next to you disappears from view. Your field of vision is shrinking. It now looks like you are seeing things through a pipe. The front corner of the car disappears from view as your peripheral vision disappears. The visual pipe's diameter is getting smaller and smaller. You sink into the seat further still as the number of g's climb further. In a flash you see black. You have just "blacked out." You are unconscious until the number of g's are reduced and the blood returns to your brain.

Amusement park owners and insurance companies don't want the previously described situation to occur. It would limit repeat riders and the number of potential consumers who can safely ride the coaster. Most roller coasters keep the g's felt under 5 g's on an inside loop or the bottom of a dip after a hill. When a rider travels over a hill at a high rate of speed, he experiences negative g's. A negative g is the multiple of a person's weight that is needed to keep a rider in his seat. Negative g's also force the coaster car to try to come up off the track. Negative g's are a rider's heaven and a designer's nightmare. Negative g's are avoided as much as possible.

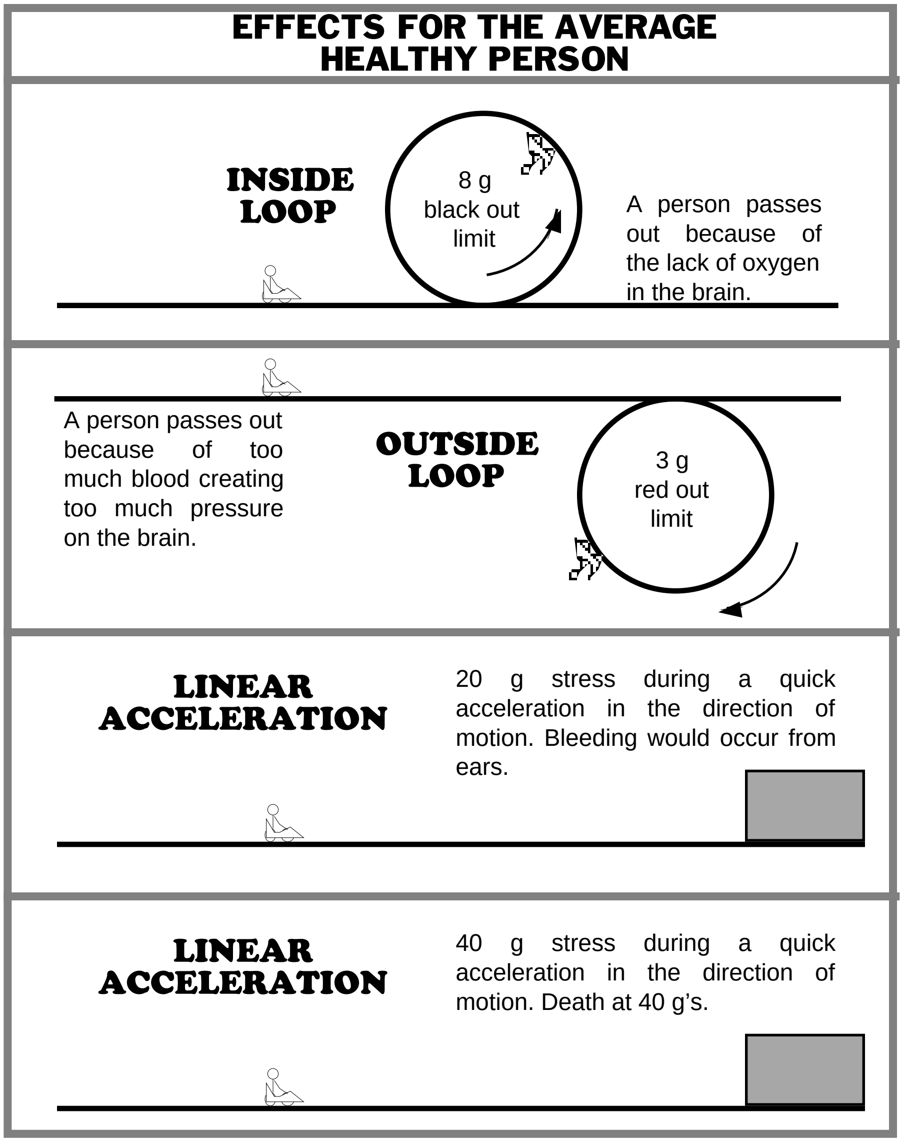
A negative g has a different effect on a rider than a positive g. Both negative and positive g's can cause a rider to pass out. But negative g's cause a rider to "red out." A red out condition occurs when there is too much pressure on the brain caused by too much blood in the head. The extra pressure can cause blood vessels to burst and kill the rider. This is a sure way to limit the number of repeat riders.

There is another way for a rider to experience negative g's. It is related to the length of the train. The roller coaster track is designed for the dynamics at the center of mass of the coaster train. Negative g's are experienced by the rider at the back of the train as he travels over a hill. For an empty train, the center of mass is in the middle of the train.Whatever speed is acquired by the center of the train is the speed for the entire train. After the center of a train passes over a hill it begins to gain velocity. As the center speeds up so does the back of the train. This means that the rear of the train will travel over the hill faster than the middle of the train. If the rider travels over the hill faster than the designed velocity of the hill the rear car will be whipped over the hill.



The rear car is traveling over the hill at 3 m/s faster than the center was. If the hill is designed for 10 m/s then the rear train car will have a tendency to leave the tracks. Under-carriage wheels will hold it to the tracks.

SOME “g” DETERMINATORS:



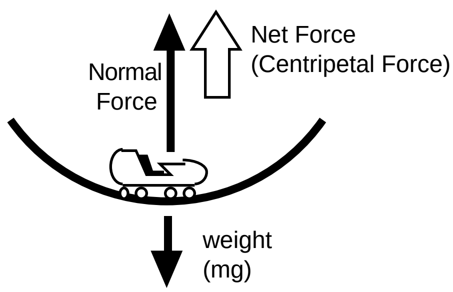
CALCULATION OF THE G'S FELT

To calculate the g's felt, a formula from circular motion will be utilized. Since energy relationships do not utilize time, the circular motion formula used will also not utilize time.

$$a_{CENTRIPETAL} = \frac{v^2}{R}$$

$$g\text{'s} = \frac{a_{CENTRIPETAL}}{9.8\ \frac{m}{s^2}}$$

Where "**v**" is the velocity of the body and "**R**" is the radius of the circle traveled. To calculate the velocity a body is traveling, use energy relationships to solve for the kinetic energy and the associated velocity. One more thing. To calculate the g's felt remember that the g's felt by the rider is the normal force on the seat of the rider divided by the mass then converted into g's. As a rider enters a loop he will feel 2 forces.



The real number of interest is the number if g's felt by the passenger traveling in the vertical circle. The g's felt are calculated below.

$\Sigma F_y = m(a_c)$ = (Normal Force) - Weight

$\Sigma F_y = mv^2/R$ = (Normal Force) - mg

(Normal Force) = $mv^2/R + mg$

recall that... (Normal Force)/mg = g's felt by the rider

thus... (Normal Force)/g = mv^2/R/mg + mg/mg

g's felt by the rider = (centripetal accelerationin g's) + 1 at the bottom

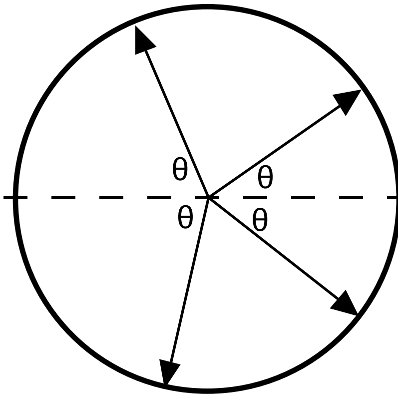
These results can be summarized as follows...

<u>ABOVE</u> the horizontal

g's FELT = g's - sin(θ)

g's FELT = g's + sin(θ)

<u>BELOW</u> the horizontal



ZERO degrees (Horizontal)

All angles are measured to the horizontal axis

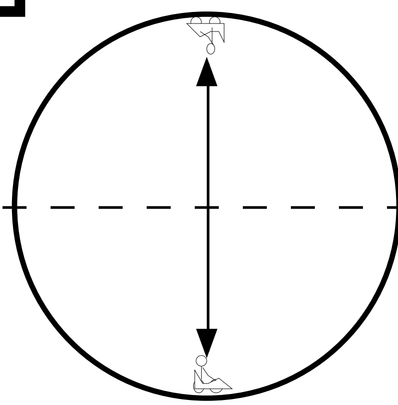
SPECIAL CASE (SHORT CUT)

<u>ABOVE</u> the horizontal

g's FELT = g's - 1

g's FELT = g's + 1

<u>BELOW</u> the horizontal



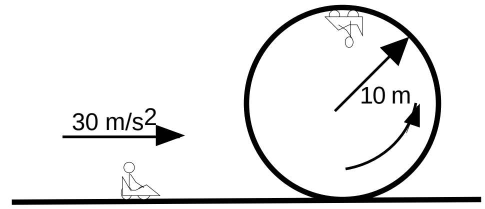
ZERO degrees (Horizontal)

All angles are measured to the horizontal axis

These results can be interpreted easily. As a rider enters the loop, the track has to exert a normal force upwards to supply the necessary centripetal force and acceleration to make the rider travel in a circle. But because the loop is vertical and the rider is at the bottom the normal force not only has to supply the centripetal force but must ***<u>also</u>*** overcome the pull of gravity. That's why 1 g is added in the equation. At the top of the loop, 1 g is subtracted from what is felt because the pull of gravity is helping the normal force exerted by the track instead of needing to be overcome.

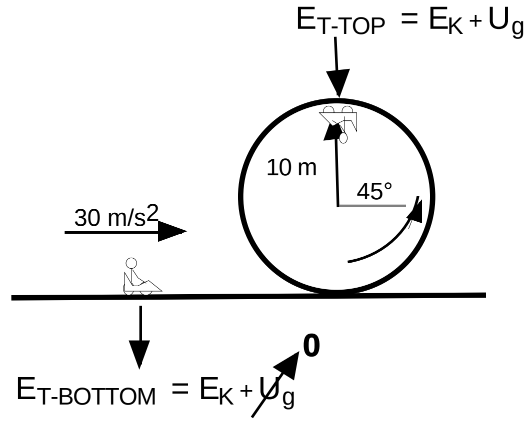
EXAMPLE 1

In a roller coaster ride a rider travels as shown to the right. How many g's will the rider feel at the top of the loop?



SOLUTION

To calculate the g's at the top of the loop, you will need to know the velocity of the rider there. To find velocity, use kinetic energy.



$$E_{T\text{-}TOP} = E_{T\text{-}BOTTOM}$$
$$E_K + U_g = E_K + 0$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$$
$$(1/2)v^2 + gh = (1/2)v^2$$
$$(1/2)v^2 + (9.8)17.07106 = (1/2)(30)^2$$
$$(1/2)v^2 + 167.2964 = (1/2)30^2$$
$$282.7036 = (1/2)v^2$$
$$565.4072 = v^2$$
$$\mathbf{v = 23.7783\ m/s}$$

$$a_{CENTRIPETAL} = \frac{(23.7783\ m/s)^2}{10} = \frac{565.4076}{10} = 56.5408\ m/s^2$$

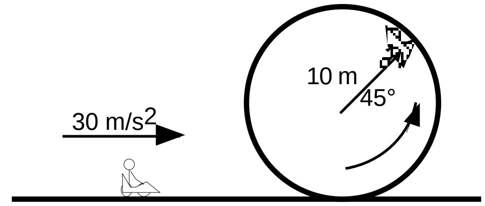
$$g\text{'s} = \frac{56.5408\ m/s^2}{9.8\ m/s^2} = 5.77\ g\text{'s}$$

g's FELT = 5.77-1 = **4.77 g's felt**

The rider will not pass out because 4.77 is less than 8 g's.

EXAMPLE 2

In a roller coaster ride a rider travels as shown to the right. How many g's will the rider feel at this location of the loop?



SOLUTION

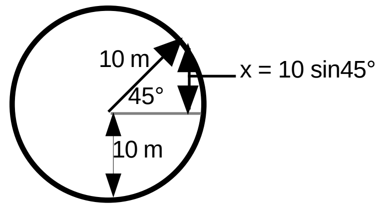
height of the roller coaster car;

$h = \text{radius} + x$

$h = 10 + 10\sin 45°$

$h = 10 + 7.07107$

$h = 17.07107\text{ m}$



$E_{T\text{-}TOP} = E_{T\text{-}BOTTOM}$

$E_K + U_g = E_K + 0$

$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$

$(1/2)v^2 + gh = (1/2)v^2$

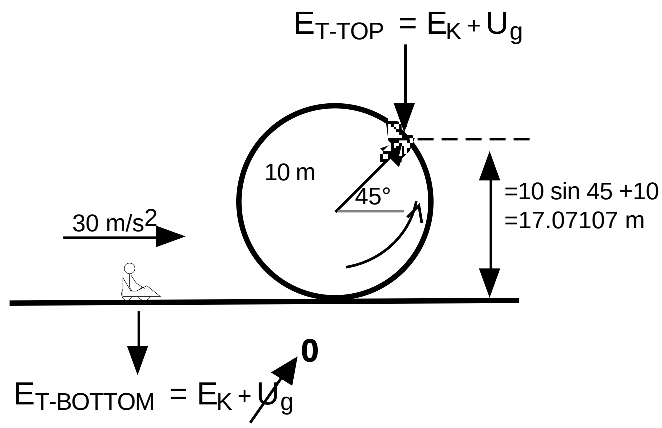
$(1/2)v^2 + (9.8)17.0711 = (1/2)(30)^2$

$(1/2)v^2 + 167.30 = (1/2)30^2$

$282.703 = (1/2)v^2$

$565.406 = v^2$

$\mathbf{v = 23.778\ m/s}$



$$a_{CENTRIPETAL} = \frac{(23.778\ m/s)^2}{10} = \frac{565.406}{10} = 56.541\ m/s^2$$

$$g\text{'s} = \frac{56.541\ m/s^2}{9.8\ m/s^2} = 5.77\ g\text{'s}$$

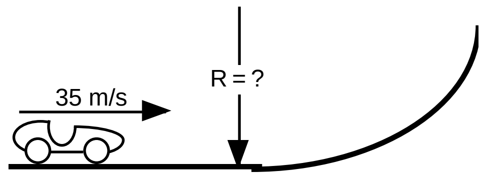
$$g\text{'s FELT} = 5.77 - \sin 45° = \mathbf{5.01\ g\text{'s felt}}$$

The rider will not pass out because 5.01 is less than 8 g's.

THE IRREGULAR LOOP EXAMPLE

The simple loop is easy enough to calculate. The irregular shaped loop needs a little more work. The velocity as the car enters the loop should be known. First establish the g's felt at the bottom. Subtract one g to know what the track exerts. Then convert these g's to m/s^2. Now solve for the radius.

EXAMPLE



STEP 1 (I'm randomly choosing 6 g's as the limit for the rider)

Therefore the centripetal acceleration of the track is 6g - 1g = 5g's.

STEP 2 (convert these g's to m/s^2)

$$(5g)\left(\frac{g}{9.80\ m/s^2}\right) = 49\ m/s^2$$

STEP 3

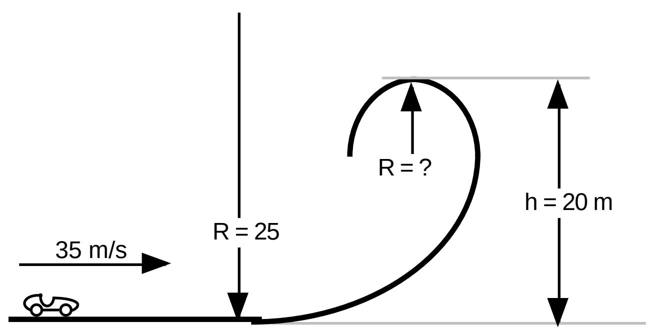
$$a = \frac{v^2}{R}$$

$$49\ m/s^2 = \frac{35^2}{R}$$

$$R = 25\ m$$

Now to calculate what the rider feels at the top of the loop.

Decide on the height of the loop. Then decide how many g's the rider will experience. Use the loop formulae with centripetal acceleration to calculate the radius.



STEP 4

The top of the loop will be at 25 m. (Chosen pretty much at random.)

STEP 5

I'm randomly choosing 6 g's again as the limit for the rider. It could be almost any number. At the top of the loop add 1 g for the centripetal force. ("Add" because the rider is upside down.)

6g's + 1 g = 7g

STEP 6 convert g' to m/s^2

$$(7g)\left(\frac{g}{9.80\ ^{m}/_{s^2}}\right) = 68.6\ ^{m}/_{s^2}$$

STEP 7

$$(^1/_2)(m)(35)^2 = (^1/_2)(m)v_o^2 + (m)(9.80\ ^{m}/_{s^2})(20\ m)$$

The m's divide out.

$$(^1/_2)(35)^2 = (^1/_2)v_o^2 + (9.80\ ^{m}/_{s^2})(20\ m)$$

$$v_o = 28.86\ ^{m}/_{s}$$

STEP 8

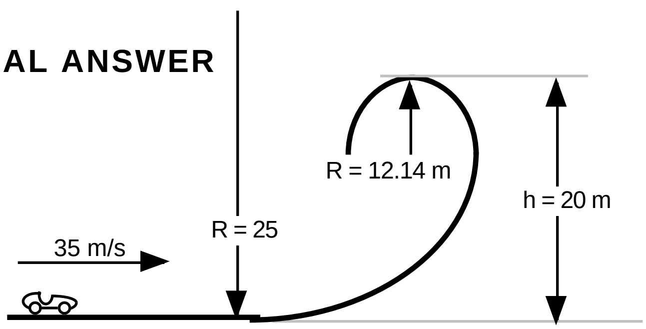
Calculate the radius at the top

$$a = \frac{v^2}{R}$$

$$68.6\ ^{m}/_{s^2} = \frac{28.86^2}{R}$$

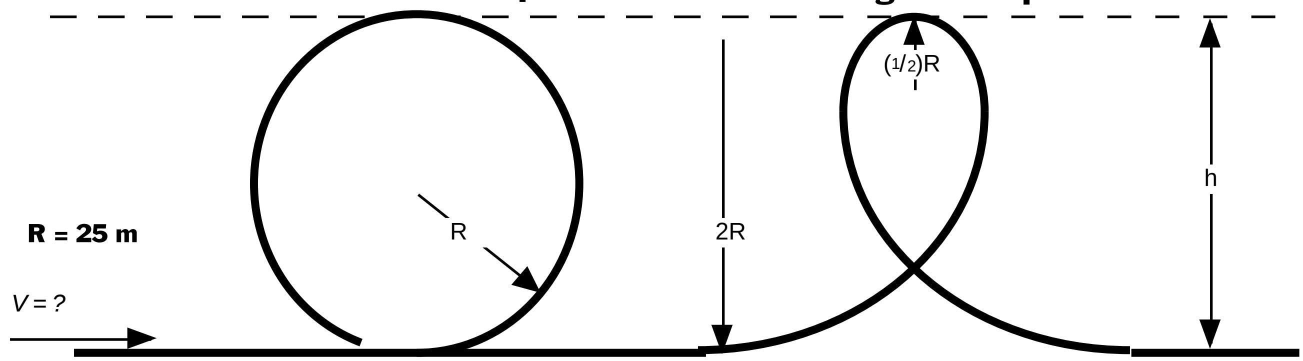
$$R = 12.14\ m$$

FINAL ANSWER



REALITY CHECK

In reality, a person will not pass out the instant he/she reaches 8 g's. It will take a few seconds of being at 8 g's for the person to pass out. But for the sake of easy calculations we will assume that the instant 8 g's is reached a person will pass out. 8 g's is an average. peaple generally pasout between 6 to 10 g's. (FYI: The 40 g mark mentioned earlier is instantaneous for death.)



The height of both loops is the same.

1 What must the velocity of the car be at the top of the circular loop such that the rider FEELS weightless at the top of the first loop?

2 What must the velocity of the car be at the bottom of the circular loop such that the rider FEELS weightless at the top of the first loop?

3 How many g's does the rider feel as he enters the circular loop, at the bottom?

4 How fast is the rider traveling when he enters the irregular loop?

5 How many g's does the rider feel as he enters the irregular loop?

6 How fast is the rider traveling at the top of the irregular loop?

7 How many g's does the rider feel at the top of the irregular loop?

NEW SET OF PROBLEMS

8 How fast must the car be traveling at the top of the klothoid loop if the rider is to experience 2.00 g's?

9 How fast would the rider be traveling as she enters the irregular loop?

10 How many g's does the rider feel as she enters the irregular loop?

11 How many g's does the rider feel as she enters the circular loop?

12 How many g's does the rider feel as she passes over the top of the circular loop?

ANSWERS

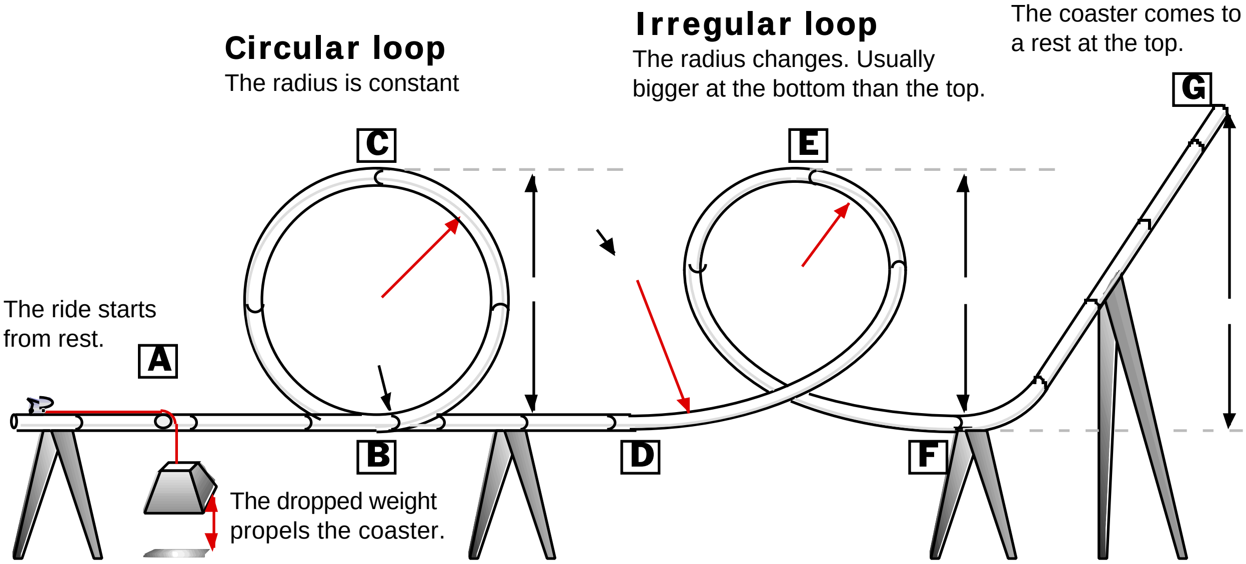
1	15.65 m/s	2	35.00 m/s	3	6 g's	4	35 m/s
5	3.5 g's	6	16.65 m/s	7	1.00 g's	8	19.17 m/s
9	36.71 m/s	10	3.75 g's felt	11	6.50 g's felt		

12 0.50 g's (He feels like he might fall out of his seat.)

The activity on the following page is good for a quick introduction to loop design. It is appropriate for students who lacks the necessary math skills. It could also be used as a quick overview to loop design.

The first page is to be used as a reference. The second page is where the calculations are done on a spreadsheet.

This diagram is to be used in conjunction with the spreadsheet below and the questions on the following page.



Below is the spreadsheet and its formulae for the "Investigating the Loop Using a Spreadsheet" handout. The text on the left hand side is in the "B" column. The "B" column is right justified.

The Spreadsheet's formulae

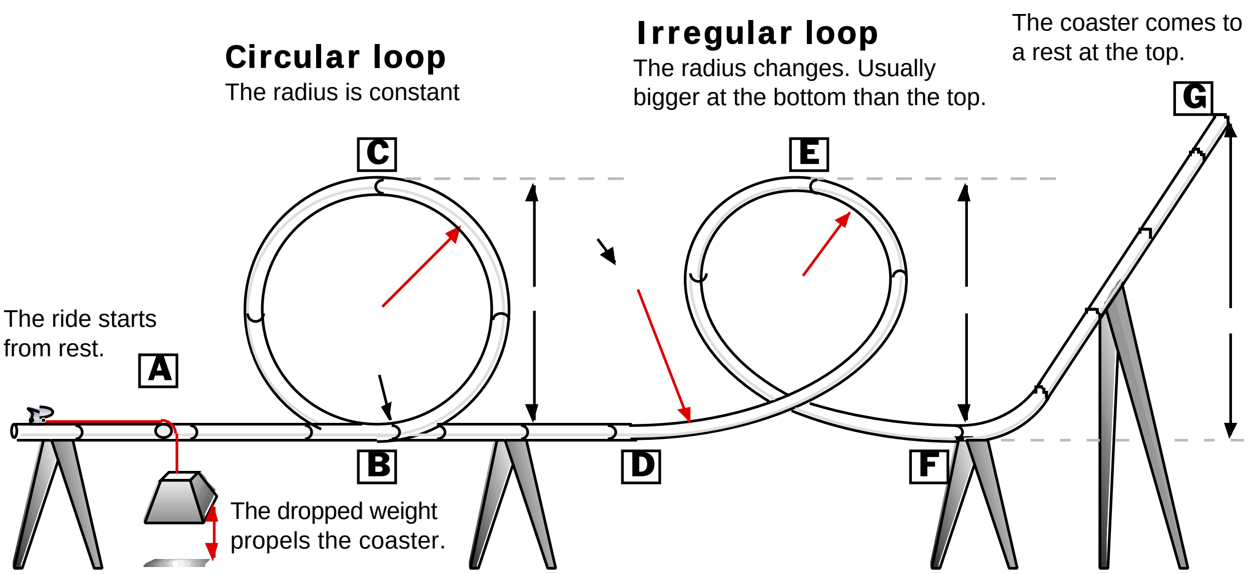
	A	B	C
1		train's mass (kg):	
2		weight's mass (kg):	
3		weight's drop height (m):	
4		Train's Acceleration (g's):	=(C2/(C3+C2))
5		Velocity at "A" (m/s):	=SQRT(2*C4*9.8*C3)
6		Velocity at "B" (m/s):	=C5
7		Radius of 1st loop (m):	
8		g's felt at "B":	=(C6*C6/C7)/9.8+1
9		Height at "C" (m):	=C7*2
10		Velocity at "C" (m/s):	=SQRT(C6*C6-(2*9.8*C9))
11		g's felt at "C"	=(C10*C10/C7)/9.8-1
12		Velocity at "D" (m/s):	=C5
13		Radius at "D" (m):	
14		g's felt at "D":	=(C12*C12/C13)/9.8-1
15		Radius at "E" (m):	
16		Height at "E" (m):	
17		Velocity at "E" (m/s):	=SQRT(C6*C6-(2*9.8*C16))
18		g's felt at "E":	=(C17*C17/C15)/9.8+1
19		Height to "G" (m):	=C12*C12/19.6

Questions:

1 A 5500 kg coaster train is propelled by a 120,000 kg weight that is dropped 20.0 m to the ground. The first loop has a radius of 25 m.
   a) How many g's are felt by the rider as he enters the first loop?
   b) How fast is the rider traveling as he travels over the top of the first loop?
   c) How many g's are felt by the rider as he travels over the top of the first loop?
   d) Make the radius at the bottom of the irregular loop 25 m. What must the radius at the top of the second loop be if its height is 42 meters?

2 A 5500 kg coaster train is propelled by a 91,000 kg weight that is dropped 25.0 m to the ground.
   a) What must the radius of the first loop be so that a rider feels 2 g's as she enters the loop?
   b) What must the radius of the second loop be so that a rider feels 2 g's as she enters the loop?
   c) How high and what radius must the irregular loop be so that a rider feels the same g's at the top and bottom?

3 Design a roller coaster where the rider feels 2.9 to 3.1 g's at every acceleration *except* at the top of the first loop. Enter your numbers at the appropriate locations on the diagram below.



4 In terms of g's felt by a rider, what are the benefits of using an irregular loop versus a circular loop?

Center of Mass

CENTER OF MASS

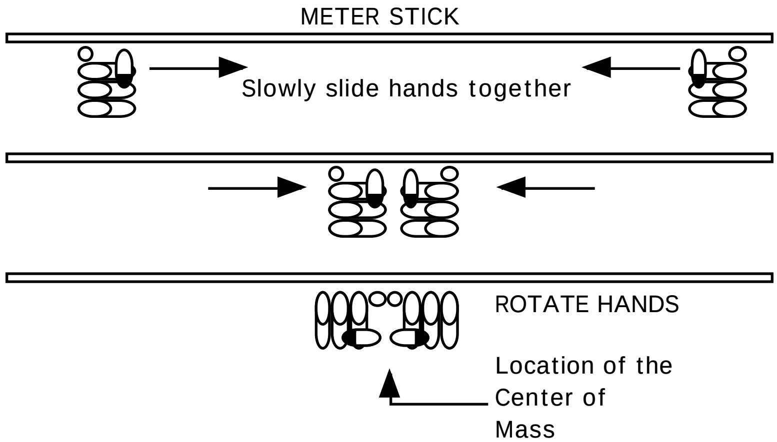
Center of Mass
The balance point of an object or collection of objects.

DEMO

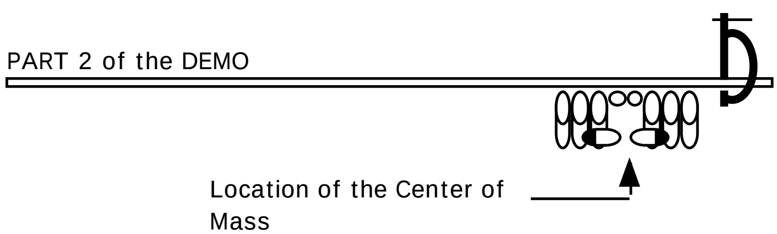
Materials: Meter stick, C-clamp

Procedure

Hold the meter stick horizontally between two fingers. Slowly slide your hands together. For a real challenge, close your eyes when sliding your hands together. Your hands will always meet under the center of mass.



Attach a c-clamp at one end of the meter stick. Redo the demonstration. Your fingers will still meet under the center of mass.

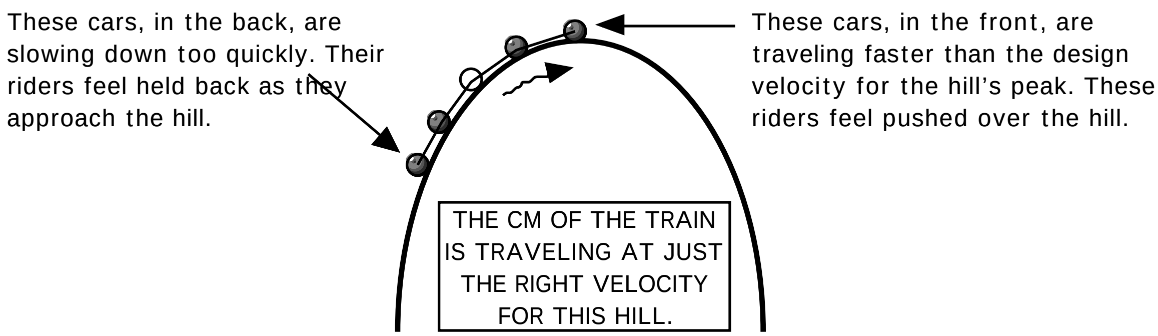


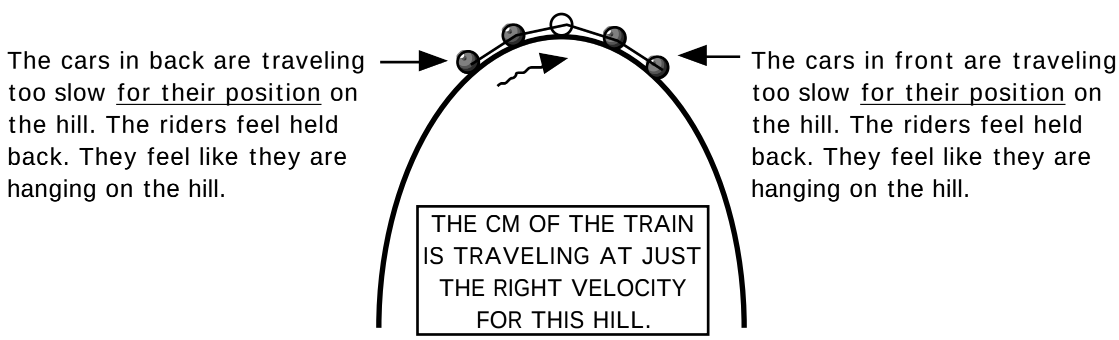
CENTER OF MASS OF A ROLLER COASTER TRAIN

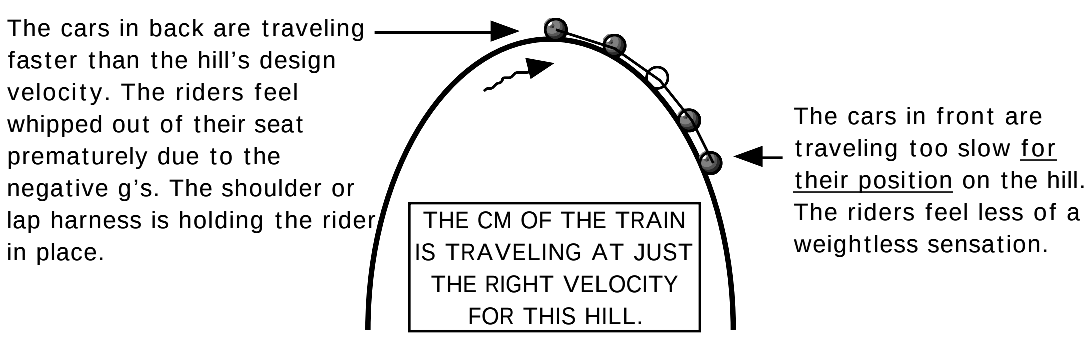
The center of mass of a train would be in the center of the train. This is assuming all the riders are of the same mass. The feeling a track is designed for is engineered around the center of mass of a train. A rider gets a different feeling if she is to ride some distance away from the center of mass.

OVER THE HILLS

A hill is designed for a specific velocity. The design velocity is chosen such that the rider located at the train's center of mass will, at most, feel weightless. The hill's shape determines the design velocity. This shape also dictates a specific velocity at each part of the hill.







This can be demonstrated by using the HotWheels™ train. (Construction of the HotWheels™ train is described on page 81.) Set up a box with a track running horizontally over the top. Slowly roll the train over the hill. As the front of the train begins to pass over the hill it will not speed up until the middle, center of mass of the train, travels over the hill.

Banked Curves

(Circular Motion, Free Body Diagrams, Kinematics and Physiological Effects)

by Tony Wayne

HORIZONTAL CURVES AND TURNS

A horizontal curve is a curve that does not rise or fall. There are two type of curves, flat curves and banked curves.

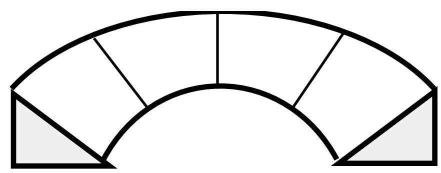
FLAT CURVES

A flat curve gives a rider the sensation of being thrown sideways. If the roller coaster car's velocity is fast enough and the radius small enough, the stresses on the car's under carriage can be tremendous. For a flat curve the inward net acceleration felt by the rider is calculated from the equation.

$$a = \frac{v^2}{R}$$

Where "a" is the acceleration felt by the rider to the inside of the circle, "v" is the velocity of the car and "R" is the radius of the curve. This acceleration can be converted to g's by dividing it by 9.80 $^{m}/_{s^2}$.

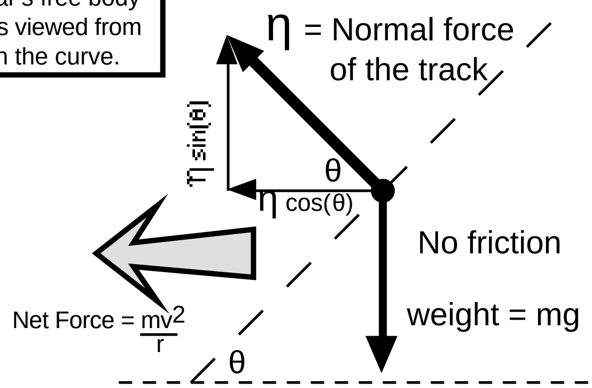
BANKED CURVES



A banked curve reduces the rider's sensation of being thrown sideways by turning the car sideways. The car is tilted. The trick is to tilt the track just the right amount.

The ideal banked curve is one where no outside forces are needed to keep the car on the track. In other words, if the banked curve were covered with ice -no friction- and the coaster did not have a steering mechanism the car would stay on the track. These are the forces acting on the car as the car travels around horizontal banked curves.

Coaster car's free body diagram as viewed from the rear on the curve.



This diagram yields the following relationships

$\Sigma F_x = mv^2/r = \eta \cos(\theta)$

$\Sigma F_y = 0 = \eta \sin(\theta) - mg$

therefore

from ΣF_x $\eta = \dfrac{mv^2}{(R)\cos(\theta)}$

from ΣF_y $\eta = \dfrac{mg}{\sin(\theta)}$

$$\frac{mg}{\sin(\theta)} = \frac{mv^2}{(R)\cos(\theta)}$$

$$\frac{g}{\sin(\theta)} = \frac{v^2}{(R)\cos(\theta)}$$

$$R = \frac{v^2 \sin(\theta)}{g\cos(\theta)}$$

$$\boxed{R = \frac{v^2 \tan(\theta)}{g}}$$

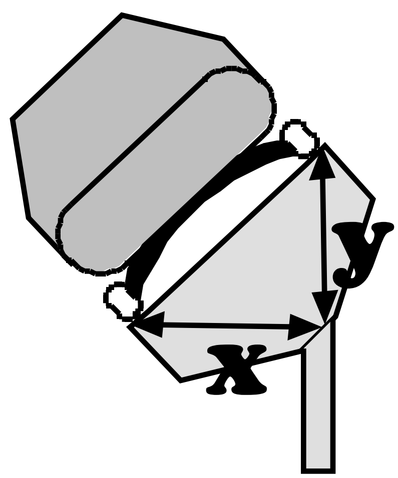
This is for the *ideal* banked curve where no friction is required to keep the car from sliding to the outside or inside of the curve. On a given curve if the velocity is greater or less than the design velocity then the cars may need a little frictional help to keep them on the track.

If your not comfortable with trigonometry functions, the equations can be rewritten and used as shown below.

$$\mathbf{a = \frac{(g)y}{x}}$$

$$\mathbf{v = \sqrt{\frac{(Rgy)}{x}}}$$

$$\mathbf{R = \frac{(x)v^2}{(g)y}}$$



The draw back to this method is in measuring the lengths of “x” and “y.”

Calculating g’s Felt on a Banked Curve

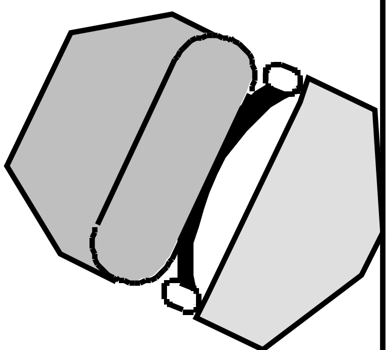
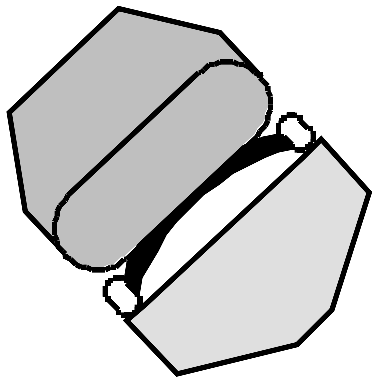
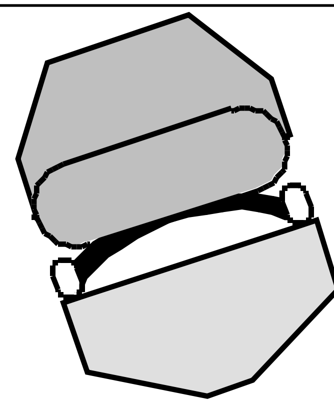
Recall that the g’s felt is equal to the normal force divided by mass and then divided by g to convert to from m/s^2 to g’s.

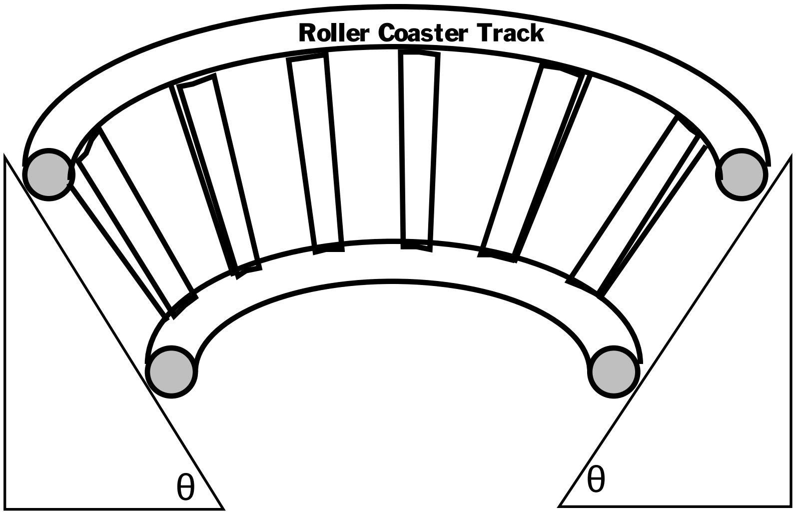
from ΣF_y $\eta = \dfrac{mg}{\sin(\theta)}$ ... from the above derivation.

g's felt $\eta = \frac{\cancel{mg}}{\sin(\theta)\cancel{mg}}$

$$\text{g's felt} = \frac{1}{\sin(\theta)}$$

Remember this is for the ideal banked curve with no friction.

HIGH SPEED BANK	MEDIUM SPEED BANK	LOW SPEED BANK
		
Too much bank for the car's velocity. the car could tip to the inside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself down. Friction is needed to keep the car on the track.	At just the right bank for the car's velocity, the car does not need any type of undercarriage to stay on the track. The rider feels a force pushing his bottom into the seat. This is the optimum position where no friction is needed to keep the car on the track.	Not enough bank for the car's velocity. The car could tip to the outside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself to the outside of the curve -sideways. Friction is needed to keep the car on the track.



$$\text{g's felt} = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{v^2}{rg}$$

θ is the angle where no outside forces other than gravity are needed to keep the car from sliding to the outside or inside of the curve.

1 What must the curve's angle be for a roller coaster car to travel around a curve of radius 30 m at 20 m/s?

2 How many g's are felt by a rider as he travels around the banked curve in the previous problem?

3 A car is to make it around a banked curve. The radius is 15.35 m and the car will travel at 30 m/s. What is the optimum banking angle of the curve?

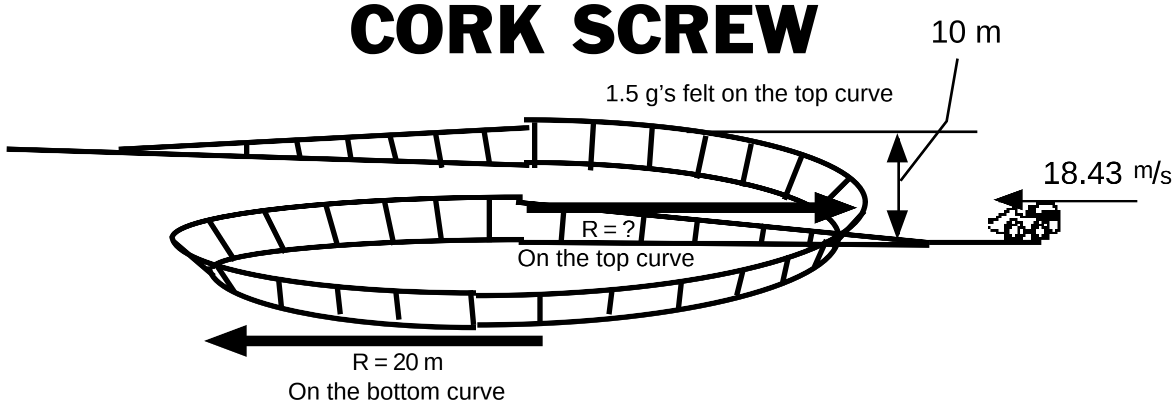
4 A car is to make it around a banked curve. The radius is 15.35 m and the car will travel at 30 m/s. This roller coaster is on the moon where the acceleration due to gravity is 1.67 m/s^2. What is the optimum banking angle of the curve?

5 A rider is to make it around a curve of radius of 24.28 m so that the rider will feel 2.50 g's. What is the angle of the banked curve?

6 A rider is to make it around a curve of radius of 31.15 m so that the rider will feel 1.64 g's. How fast must the rider be traveling?

7 A rider is to make it around a curve of radius of 51.15 m so that the rider will feel 4.52 g's. How fast must the rider be traveling?

CORK SCREW



8 What is the banked angle of the bottom curve?

9 How many g's are felt by the rider along the bottom curve?

10 What is the optimum angle of the top banked curve after spiraling up 10 m?

11 What is the radius of the top curve?

ANSWERS

1	53.68°	2	1.69 g's	3	80.51°	4	88.37°
5	66.42°	6	19.92 m/s (52.69°)	7	47.01 m/s (77.22°)	8	60.01°
9	2.000 g's	10	48.19°	11	13.11m (11.99 m/s at the top)		